ELECTROMAGNETIC WAVES

What is light? This question puzzled human being for ages, but no answer emerged until electricity and magnetism were unified into ELECTROMAGNETISM by the Maxwell's equations.

These equations show that: a time varying $\vec{B}$ acts as a source of $\vec{E}$ and vice-versa, a time varying $\vec{E}$ acts as a source of $\vec{B}$. These $\vec{E}$ and $\vec{B}$ can sustain each-other forming an electromagnetic wave that propagates through space. Visible light, waves produced by TV and radio stations, X ray machines, radioactive γ ... are few examples of electromagnetic waves. They differ only in their frequency and wavelength.

Propagation of electromagnetic waves do not require a material medium: the light coming from the stars propagate in the empty space.

1 Maxwell's equations and electromagnetic waves

When the fields do not vary in time, such as: electric fields produced by charges at rest and magnetic fields produced by steady current, we can analize them independentlly, without considering any interaction between them.

However, when $\vec{B}$ and $\vec{E}$ vary in time, they are not anymore independant. When either $\vec{E}$ or $\vec{B}$ varies in time a field of the other kind is induced in the adjacent regions. The perturbance constituted by time varying $\vec{E}$ and $\vec{B}$ can propagate in space from one region to another without need of matter as ELECTROMAGNETIC WAVE.

Electricity, Magnetism and light.

Maxwell proved in 1865 that an electromagnetic wave propagates in free space with the speed of light. This indicated that light should have electromagnetic origin.

Maxwell also discovered that the basic principles of electromagnetism can be expressed in terms of four equations (Maxwell's equations):

(1) Gauss law for electric fields:
$$\boxed{\oint \vec{E}\cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}}$$

(2) Gauss law for magnetic fields, showing the absence of magnetic monopoles
$$\boxed{\oint \vec{B}\cdot d\vec{A} = 0}$$

(3) Ampère's law including displacement current
$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i_c + \varepsilon_0 \frac{d\Phi_E}{dt}\right)}$$

(4) Faraday's law
$$\boxed{\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}}$$

valide for $\vec{E}$ and $\vec{B}$ in vacuum. If material is present $\varepsilon_0 \to \varepsilon = \varepsilon_0 K$

$\mu_0 \to \mu = \mu_0 \cdot K_m$

According to Maxwell's equations:

(1) a point charge at rest produces a static $\vec{E}$ but no $\vec{B}$

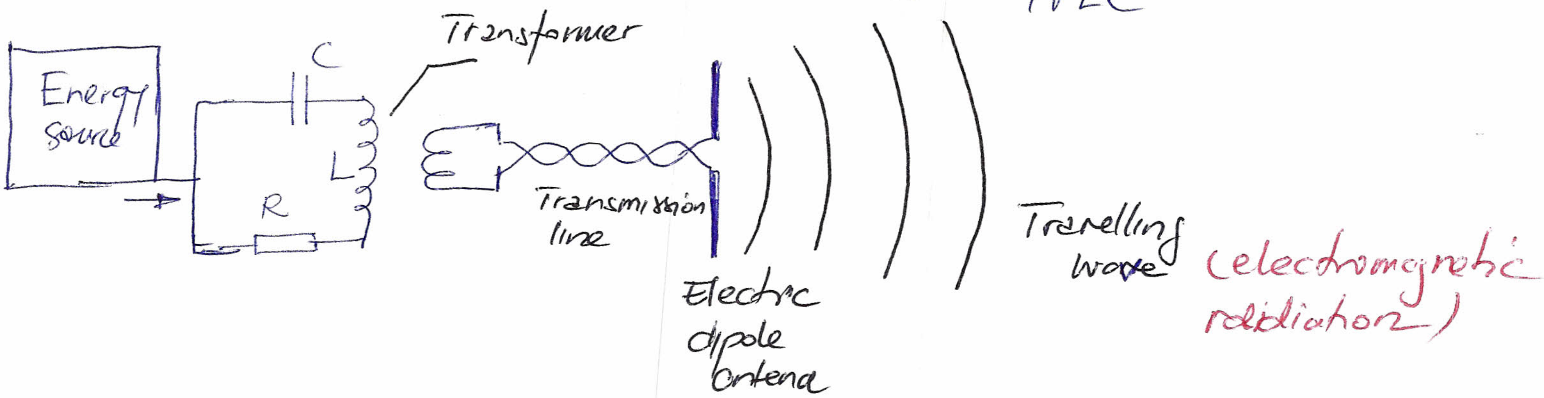
(2) a point charge moving with constant velocity produces both $\vec{E}$ and $\vec{B}$ fields.

Maxwell's equations can be used to show that in order for a point charge to produce electromagnetic waves the charge must accelerate. It's a result of Maxwell's equations that every accelerated charge radiates electromagnetic energy.

Generating electromagnetic radiation

One way in which a point charge can be made to emit electromagnetic waves is to make it oscillate in simple armonic motion. Remember the oscillating RLC circuit (or LC ideal oscillator) in which $q(t) = q_0 \cos(\omega t + \varphi)$

$\omega = 1/\sqrt{LC}$



Antena has the role of an electric dipole, being constituded of two thin conducting rods. The oscillatory current in the oscillator (RLC) causes charge to oscillate sinusoidally in the rods of antena at the angular frequency $\omega \doteq 1/\sqrt{LC}$. $\Rightarrow$ electric dipole moment varying sinusoidally in time $\Rightarrow \vec{E}(t) \longrightarrow \vec{B}(t)$ . . . .

In a radio receiver : the antena is also a conductor; the fields of wave emitted by a distant transmitter exert forces on free charges within the receiver antena producing an oscillating current that is detected and amplified by the receiver circuitry.

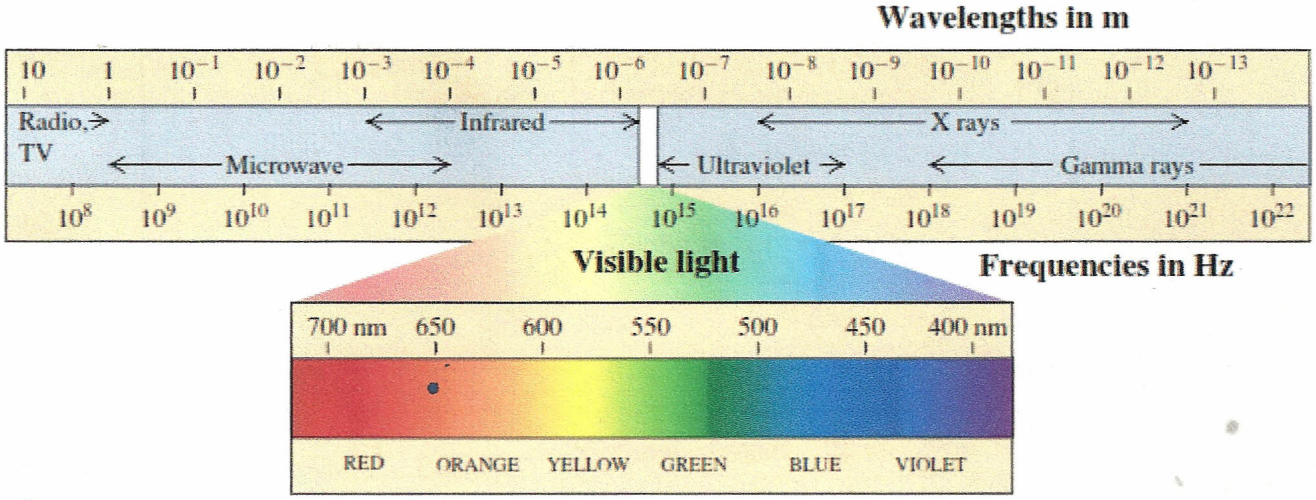
The electromagnetic spectrum

The electromagnetic spectrum encompasses electromagnetic waves of all frequencies and wavelengths

Despite different production mechanisms and uses, they all propagate in vacuum with same speed:

$$\boxed{c = 299\,792{,}458\ m/s} \qquad (\cong 3 \cdot 10^8\ m/s)$$

$$\boxed{c = \lambda \nu}$$



550 nm * maximum sensitivity of eye (human)

VISIBLE SPECTRUM of LIGHT.

Ordinary white light includes all visible wavelengths. Using special sources or filters, we can select only a narrow band of wavelengths, within a range of few nm. Such source of light is MONOCHROMATIC (single color) light.

LASERS produce almost monochromatic light.

Examples:

AM radio waves: $f = 5.4 \cdot 10^{5} - 1.6 \cdot 10^{6}\ Hz$

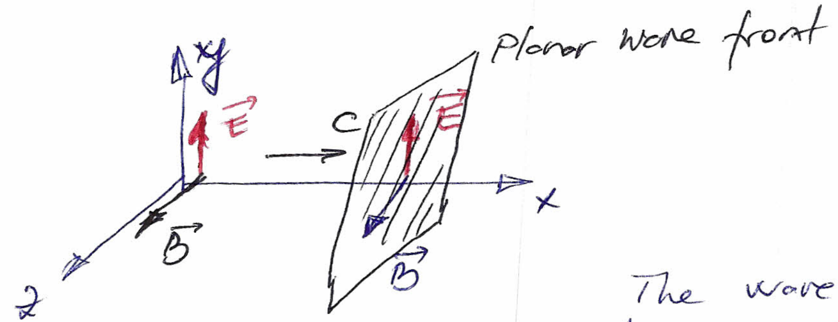
FM radio waves $f = 8.8 \cdot 10^{7} - 1.08 \cdot 10^{8}\ Hz$

Microwaves (cell phones) $f \sim 3 \cdot 10^{9}\ Hz$

Cameras with IR, UV... vision

2) Plane electromagnetic waves and the speed of light.

We assume an electric field that has only an y component and a magnetic field with only z component and both move together with the velocity +c along the axis +x.



The wave front propagates with the speed c.

We verify that our wave satisfies Maxwell's equations. From calculations we can show that:

(1) in order to satisfy 1st and 2nd Maxwell equations (Gauss law for $\vec{E}$ and $\vec{B}$) the electric and magnetic field have to be perpendicular to the direction of propagation; that is, the wave must be TRANSVERSE.

(2) to satisfy the Faraday's law we should have:

$$E = cB$$ (electromagnetic wave in vacuum)

(3) to satisfy the Ampère's law

$$B = \varepsilon_0 \mu_0 c E$$

=> to satisfy both Faraday and Ampère's law

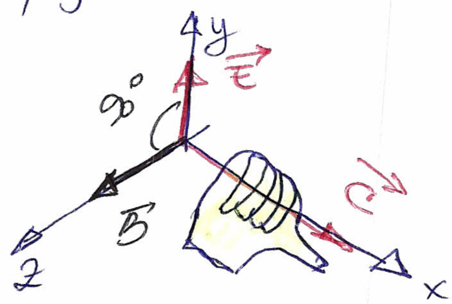
$$\varepsilon_0 \mu_0 c^2 = 1 \quad \Rightarrow \quad \boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}}$$

$$c = \frac{1}{\sqrt{8,85 \cdot 10^{-12} \cdot 4\pi \cdot 10^{-7}}} = 3 \cdot 10^8 \, m/s$$

(which is the speed of light and EM. waves in vacuum)

Key properties of electromagnetic waves

(1.) the wave is transverse. Both $\vec{E}$ and $\vec{B}$ are perpendicular to the direction of propagation. $\vec{E} \perp \vec{B}$ and the direction of propagation is the direction of $\vec{E} \times \vec{B}$



(2) There is a definite ratio between magnitudes of $\vec{E}$ and $\vec{B}$

$$\boxed{E = cB}$$

(3) The wave propagates in vacuum with definite and unchanged speed c

(4) Unlike mechanical waves which need a medium to propagate (water, air..) the electromagnetic waves require no medium.

Electromagnetic waves have the property of <u>POLARIZATION</u>. The choice of y direction for $\vec{E}$ was arbitrary. If we would choose $\vec{E}$ along z, $\vec{B}$ will be along $-y$ ($\vec{B} \perp \vec{E}$). A wave in which $\vec{E}$ is parallel to a certain axis is said to be <u>linearly polarized</u> along that axis

<u>Obs</u>: Any general wave travelling in the x-direction can be represented as a <u>superposition</u> of waves linearly polarized waves along y and z directions

Equation of electromagnetic waves

We write without demonstration the electromagnetic wave equation in vacuum:

$$\begin{cases} \dfrac{\partial^2 E_y(x,t)}{\partial x^2} = \varepsilon_0 \mu_0 \dfrac{\partial^2 E_y(x,t)}{\partial t^2} \\[2ex] \dfrac{\partial^2 B_z(x,t)}{\partial x^2} = \varepsilon_0 \mu_0 \dfrac{\partial^2 B_z(x,t)}{\partial t^2} \end{cases}$$

Same shape as wave equation for mechanical waves:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

and from comparing $\Rightarrow$ $\boxed{v = c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}}$

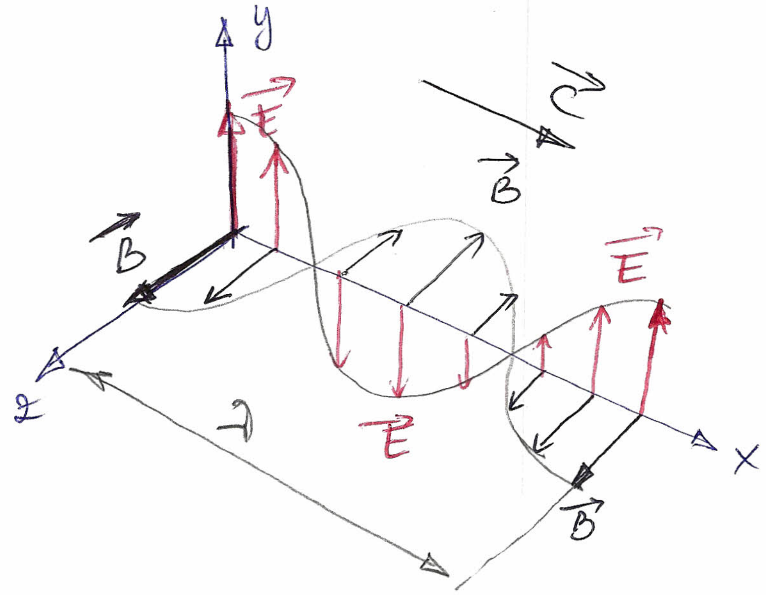
(3) Sinusoidal Electromagnetic waves

The sinusoidal electromagnetic waves are directly analogous to sinusoidal transverse waves on stretched strings (see mechanics)

In a sinusoidal electromagnetic wave, in any point of space $\vec{E}$ and $\vec{B}$ are sinusoidal functions of time and at any instant t the spatial variation of fields is sinusoidal

Using the description with plane waves, we can write: by analogy with mechanics transverse waves $y(x,t) = A\cos(kx - \omega t)$ propagation along $+x$

$\boxed{E_y(x,t) = E_{max}\cos(kx - \omega t)}$, $\boxed{B_z(x,t) = B_{max}\cos(kx - \omega t)}$



Representaton of one wavelength at $t=0$

vector form:

$$\begin{cases} \vec{E}(x,t) = \vec{j}\, E_{max} \cos(kx - \omega t) \\ \vec{B}(x,t) = \vec{k}\, B_{max} \cos(kx - \omega t) \end{cases}$$

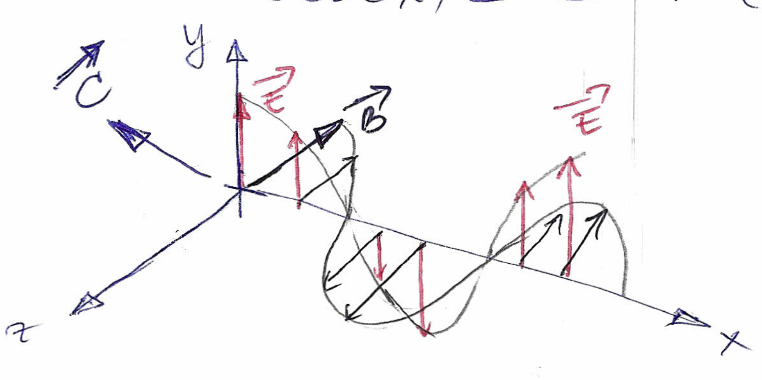
linearly polarized Eelectromagn. wave traveling along +x direction with c.

As time increases from $t=0$, the waves will travel to the right with the speed c. The above equations show that at any point the sinusoidal oscillations of $\vec{E}$ and $\vec{B}$ are IN PHASE and

$$E_{max} = c\, B_{max}$$

Obs: If the wave propagates along negative x

$$\begin{cases} E_y(x,t) = E_{max}\,(kx + \omega t) \\ B_z(x,t) = -B_{max}\,(kx + \omega t) \end{cases}$$



Electromagnetic waves in matter

Electromagnetic waves are not propagating only in vacuum, they propagate also in matter, e.g. in non-conducting materials such as dielectrics.

In a dielectric, the wave speed is not the same as in vacuum, and we denote it with v

Following the same procedure as in vacuum we get

$\boxed{E = vB}$ and $\boxed{B = \varepsilon\mu v E}$ with

from Faraday's and Ampère's laws

$\mu = \mu_0 K_m$
$\varepsilon = \varepsilon_0 K$

K_m = relative permeability of dielectric

K = dielectric permittivity

$$\Rightarrow \boxed{v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{KK_m}}\frac{1}{\sqrt{\varepsilon_0\mu_0}}}$$

speed of electromagnetic waves in a dielectric.

The ratio $\boxed{\frac{c}{v} = n = \sqrt{KK_m}}$

is known in optics as the index of refraction n

Obs 1) v is allways less than c

Obs 2) In the above equations the values of K and K_m are not those tabulated for constant fields. When $\vec{E}$ oscillates rapidly there is no time for reorientation of electric dipoles as in steady fields $\Rightarrow$ K with rapid fields is much smaller than K static.

$\boxed{K = \text{function of frequency}}$ (dielectric function)

$K = 80$ (water, steady fields) $K = 1.8$ (water, visible light)

4) Energy and momentum in electromagnetic waves

Waves carry energy. (think at energy of the sun's radiation, microwave ovens, radio-transmitters...)

Combining energy densities deduced previously for electric and magnetic fields we get, for a region of empty space where $\vec{E}$ and $\vec{B}$ are present:

$$\boxed{u = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}}$$

and $B = \frac{E}{c} = \sqrt{\varepsilon_0 \mu_0}\, E$ $\Bigg\} \Rightarrow$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}\varepsilon_0 \mu_0 E^2 = \varepsilon_0 E^2$$

$$\Rightarrow \boxed{u = \varepsilon_0 E^2}$$

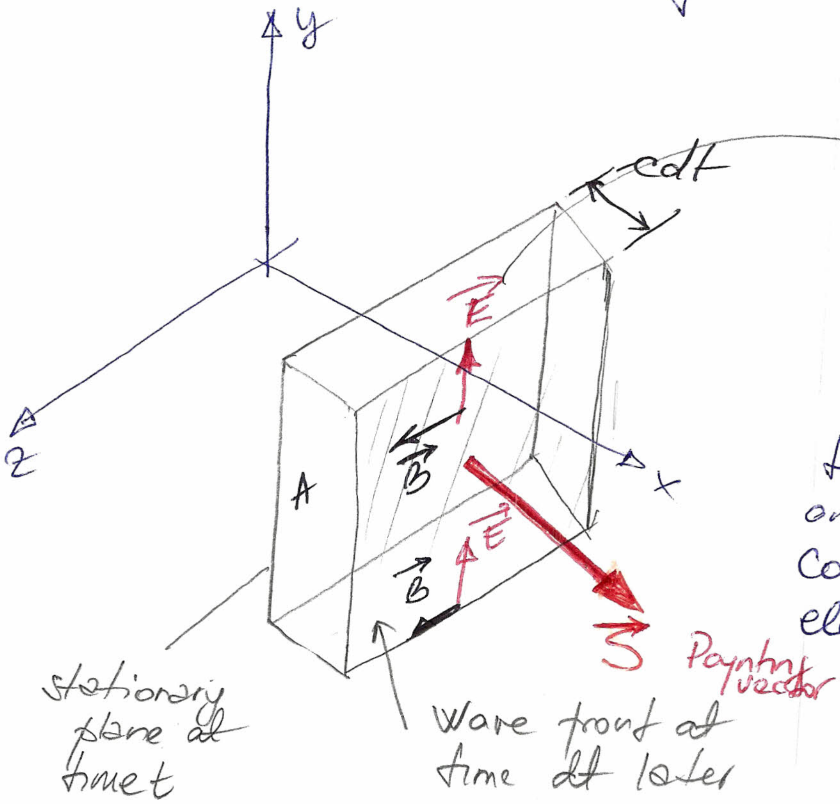
This shows that in vacuum, the energy density associated with $\vec{E}$ field is equal to the energy density of $\vec{B}$ field

Electromagnetic energy flow and POYNTING VECTOR

Electromagnetic waves we have studied up to now are travelling waves that transport energy from one region to another. We can describe this energy transfer in terms of energy transfered per unit time per unit cross-sectional area $\Longleftrightarrow$ power per unit area for an area perpendicular to the direction of wave travel.

Consider a stationary plane perpendicular to x axis that coincides with the wave front at instant t. In the time interval dt the wave front traveled with $c\,dt$

Consider an area A of the stationary plane



→ volume $Acdt$

The wave travels with c along +x.

At time dt, the volume between the stationary plane and the front of wave contains an amount of electromagnetic energy $dU = u\,A\,c\,dt$

This energy passes through the Area A in time dt.
The energy flow per unit time per unit area, denoted by S, is:

$$S = \frac{1}{A}\frac{dU}{dt} = \frac{(\varepsilon_0 E^2)\cancel{(Ac\,dt)}}{\cancel{A\,dt}}$$

$$S = \varepsilon_0 c E^2 \quad ; \quad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad ; \quad E = cB$$

$$\Rightarrow \boxed{S = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,E^2 = \frac{EB}{\mu_0}} \qquad [S]_{SI} = \frac{W}{m^2}$$

We can define a <u>vector quantity</u> which describes both magnitude and direction of the flow-rate:

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}}$$

POYNTING VECTOR IN VACUUM

introduced by the British physicist John Poynting.

Obs:

The direction of $\vec{S}$ is in the direction of propagation of the wave.

Since $\vec{E} \perp \vec{B}$ the magnitude of S is: $\boxed{S = \frac{EB}{\mu_0}}$

The total energy flow per unit time (Power P) out of any closed surface is the integral of $\vec{S}$ over the surface:

$$\boxed{\vec{P} = \oiint_A \vec{S} \cdot d\vec{A}}$$

Power = fluxul vectorului Poynting

Obs: For sinusoidal waves, as well for other complex waves, the $\vec{E}$ and $\vec{B}$ in any point vary with time => the Poynting vector is also a function of time. Because the frequency of electromagnetic waves is high, the ~~tyme~~ time variation of $\vec{S}$ is very rapid so it is more appropriate to look at its average value.

The amplitude of the average value of $\vec{S}$ at a point is called INTENSITY of THE RADIATION at that point.

For a sinusoidal wave:

$$\vec{S}(x,t) = \frac{1}{\mu_0} \vec{E}(x,t) \times \vec{B}(x,t) =$$

$$= \frac{1}{\mu_0} \left[\vec{j}\, E_{max} \cos(kx - \omega t)\right] \times \left[\vec{k}\, B_{max} \cos(kx - \omega t)\right]$$

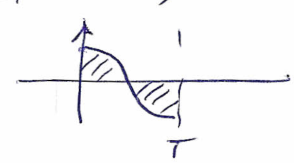
$$\vec{j} \times \vec{k} = \vec{i}$$

$$\vec{S}(x,t) = \vec{i}\,\frac{E_{max} B_{max}}{\mu_0} \cos^2(kx - \omega t)$$

The x component of the Poynting vector is:

$$S_x(x,t) = \frac{E_{max} B_{max}}{\mu_0} \cos^2(kx-\omega t) = \frac{E_{max} B_{max}}{2\mu_0} \cdot \left[1 + \cos 2(kx - \omega t)\right]$$

The time average of $\cos 2(kx-\omega t) = 0$



$\langle \cos x \rangle_T = 0$

$$\Rightarrow \quad S_{av} = \frac{E_{max} B_{max}}{2\mu_0}$$

over a full cycle the average of the Poynting vector.

using $E_{max} = B_{max} c$

$\varepsilon_0 \mu_0 = \frac{1}{c^2}$

$$\Rightarrow \quad \boxed{I = S_{av} = \frac{E_{max} B_{max}}{2\mu_0} = \frac{E_{max}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_{max}^2 = \frac{1}{2}\varepsilon_0 c E_{max}^2}$$

intensity of a sinusoidal wave in vacuum

for any other medium (dielectric)

$\varepsilon_0 \rightarrow \varepsilon = \varepsilon_0 K$

$\mu_0 \rightarrow \mu = \mu_0 K_m$

Remarkably, the energy densities for $\vec{E}$ and $\vec{B}$ turn out to be equal, even for dielectrics.

Electromagnetic moment flow and radiation pressure

We saw that electromagnetic waves carry energy. It can be shown that they also carry momentum p with a corresponding momentum density: (momentum dp / unit volume dV)

$$\boxed{\frac{dp}{dV} = \frac{EB}{\mu_0 c^2} = \frac{S}{c^2}}$$

Obs: This momentum is a property of the field, it is not associated with the mass of a moving particle in the usual sense.

There is also a corresponding momentum flow rate. The volume dV occupied by the electromagnetic wave (speed c) that passes through an area A in time dt is $dV = A c\, dt$. Substituting this in the above eq. of $\frac{dp}{dV}$ and rearanging we get the momentum flow rate per unit area:

$$\frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$

If we replace S by $\langle S \rangle \Rightarrow S_{av} = I$

This momentum is responsible for radiation pressure

$$\frac{dp}{dt}\frac{1}{A} = \frac{\text{force}}{\text{surface of absorbing area}}$$

$$\boxed{P_{rad} = \frac{S_{av}}{c} = \frac{I}{c}}$$

radiation pressure, wave totally absorbed

If the wave is totally reflected, the momentum change is twice as great

$$\Rightarrow \boxed{P_{rad} = \frac{2S_{av}}{c} = \frac{2I}{c}}$$

radiation pressure, wave totally reflected.

Example

The value of $\dot{I}$ (or S_{av}) for direct sun light is, before passing through the Earth's atmosphere, $\simeq 1{,}4$ KW/m²

$$\Rightarrow \quad P_{rad} = \frac{\dot{I}}{c} = \frac{1{,}4 \cdot 10^3 \, W/m^2}{3 \cdot 10^8 \, m/s} = 4{,}7 \cdot 10^{-6} \, Pa$$ for a totally absorbing surface

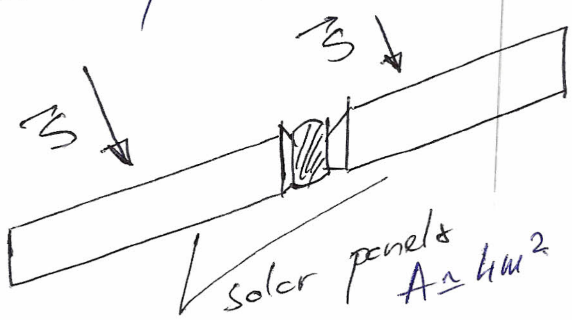
$$\text{or} \quad P_{rad} = \frac{2i}{c} = 9{,}4 \cdot 10^{-6} \, Pa$$ for a totally reflecting surface

small pressures $\simeq 10^{-10}$ atm but they can be measured by sensitive instruments.

Obs:

(1) The radiation pressure of sunlight is much greater inside the sun.

(2) Sollar panels on a satellite : total $A = 4m^2$



A sun sensor keeps the panels facing the sun

The average radiation pressure force

$$F = P_{rad} \cdot A = 4{,}7 \cdot 10^{-6} \frac{N}{m^2} \, 4m^2 = \underline{1{,}9 \cdot 10^{-5} \, N}$$

very small, but over time substantial effect can appear over the orbit of the satellite $\Rightarrow$ The effect of the radiation pressure has to be considered.

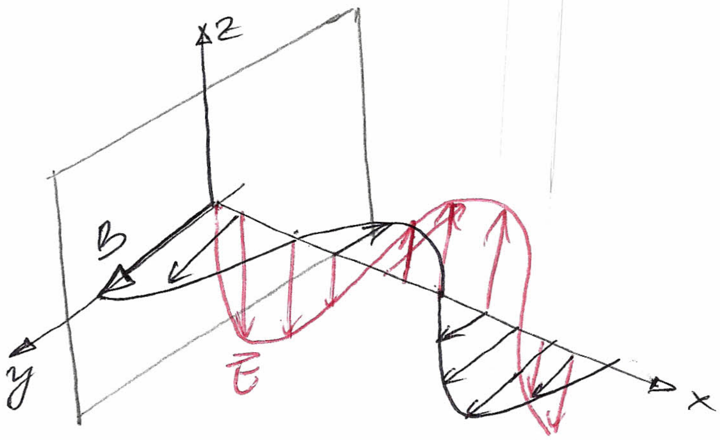
(5) Standing electromagnetic waves

Electromagnetic waves can be reflected when they met a medium with different K. (e.g. the surface of a conductor or a dielectric can serve as reflector).

The superposition principle holds for electromagnetic waves as for $\vec{E}$ and $\vec{B}$ fields.

The superposition of an incident wave and a reflected wave form a STANDING WAVE — situation analogous to standing waves in a string.

Suppose a sheet of perfect conductor (zero resistivity) is place in the plane yz, and a linearly polarized electromagnetic wave travelling along −x direction, strikes it.



Because $\vec{E}$ cannot be transverse on a conductor ⟹ net $\vec{E}$ is zero everywhere in the yz plane. The incident $\vec{E}$ was not zero but incident waves induce oscillating current on the surface of the conductor which give rise to additional electric fields so that the net vector sum of these $\vec{E}$ fields to become zero.

The currents induced on the surface of the conductor also produce a reflected wave that travels along the +x direction.

The superposition principle $\Rightarrow$

$$E_y(x,t) = E_{max}\left[\cos(kx+\omega t) - \cos(kx-\omega t)\right]$$
$$B_z(x,t) = B_{max}\left[-\cos(kx+\omega t) - \cos(kx-\omega t)\right]$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\Rightarrow \boxed{E_y(x,t) = -2E_{max}\sin kx \sin \omega t}$$
$$\boxed{B_z(x,t) = -2B_{max}\cos kx \cos \omega t}$$

analogous with stationary waves in stretched string

We see that at $x=0$ $\boxed{E_y(x=0,t)=0}$ required by the nature of ideal conductor

$\Updownarrow$

fixed point at the end of string.

Furthermore $E_y(x,t)=0$ in those planes $\perp$ to x axis for which

$\sin kx = 0$; $kx = 0, \pi, 2\pi, \ldots$

Since $k = \frac{2\pi}{\lambda}$ $\Rightarrow$ $\boxed{x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots}$

nodal planes of $\vec{E}$

equivalent to nodes of standing waves in a string.

Midway between two adjacent nodal planes $\Rightarrow$

$\sin(kx) = \pm 1$ $\Rightarrow$ $E = \pm 2E_{max}$

correspond to antinodal planes

$\Leftrightarrow$ (antinodes waves on a string)

Analyzing $b_z(x,t) = -2B_{max}\cos(kx)\cos\omega t$

$b_z(x,t) = 0 \Rightarrow \cos kx = 0 \Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4} \ldots$

nodal planes of $\vec{B}$

There are antinodal planes of $\vec{B}$ at midway between two nodal planes.

Obs ① The $\vec{B} \neq 0$ at the conductor surface. This is because the surface currents that appear to make $\vec{E} = 0$ at the surface cause magnetic field at the surface.

② The nodal planes of each field are separated by $\frac{\lambda}{2}$.

③ Nodes of $\vec{E}$ coincide with AN of $\vec{B}$ and vice-versa.

The total electric field is a sine function of t and the total magnetic field is a cosine function of t
$\Rightarrow$ the sinusoidal variations of the two fields are 90° out of phase at each point.

when $E = 0$; $B = B_{max}$
$B = 0$; $E = E_{max}$

This is in contrast with a propagative electromagn. wave where $\vec{E}$ and $\vec{B}$ are in phase.

Standing waves in a cavity

<=> normal modes in a string for mechanical waves.

Let us now insert a second conducting plane, at distance L from it, along +x axis. <=> stretched string fixed at $x=0$ and $x=L$.

Both conducting planes have to be nodal for $\vec{E}$, standing wave can exist only when the second conducting plane is placed at one of the positions where $E_y(x,t)=0$ so $L = n\frac{\lambda}{2}$ n = integer 1, 2, ...

$$\Rightarrow \boxed{\lambda_n = \frac{2L}{n}} \qquad n = 1, 2, 3, \ldots$$

The corresponding frequencies are:

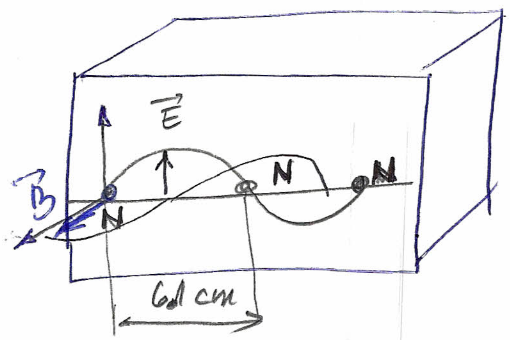
$$\boxed{f_n = \frac{c}{\lambda_n} = n\frac{c}{2L}} \qquad n = 1, 2, 3, \ldots$$

Thus there is a set of normal modes, each with a characteristic frequency, wave shape and node pattern. By measuring the nodes positions, we can measure the wavelength. Knowing its frequency, the wave speed can be determined (so ~~done~~ by H. Hertz in 1880 in pioneering investigations of electromagnetic waves)

Obs: Conducting surfaces are not the only reflectors of electromagnetic waves. Reflexions occur at interfaces between two insulating materials with different dielectric or magnetic properties. See the wave (light) reflexion, refraction phenomenon.

Example

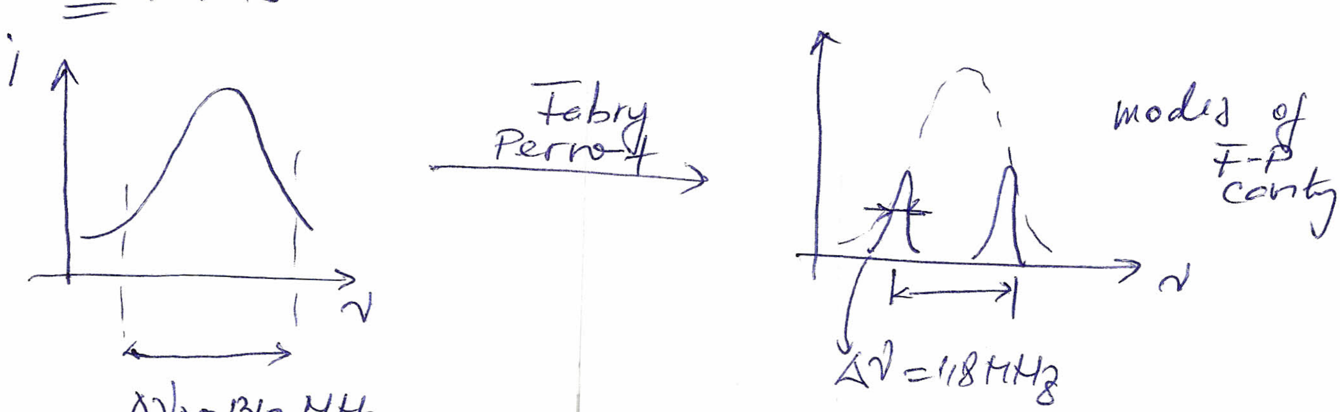
① Typical microwave oven setup sets-up a standing electromagnetic wave with $\lambda = 12{,}2$ cm ($f = 2{,}45$ GHz) a wavelength that is strongly absorbed by the water in food. Because the nodes are spaced $\frac{\lambda}{2} = 6{,}1$ cm apart, the food must be rotated while cooking. otherwise, the portion that lies at a node – where the amplitude of electric field is zero – will remain cold..



(remember the paragraph about heating food with microwaves from 1st chapter (Charge. Field. Gauss law))

② Rezonant cavities can be used to select a narrow monochromatic wave from broad spectrum, corresponding to modes (stationary modes of cavity)

ex: LASER



moving emitting atoms with Boltzmann distribution of velocities

see. Truesondal
Mecanica cuantică prin aplicații. Ed. UTPRES 2013.